

# Wilsonian Dark Matter in String Derived $Z'$ Model

L. Delle Rose<sup>♠</sup>, A.E. Faraggi<sup>♣</sup>, C. Marzo<sup>◇</sup> and J. Rizos<sup>♡</sup>

<sup>♠</sup>*School of Physics and Astronomy,  
University of Southampton, Southampton SO17 1BJ, UK  
Department of Particle Physics,  
Rutherford Appleton Laboratory, Chilton, Didcot, OX11 0QX, UK*

<sup>♣</sup>*Department of Mathematical Sciences,  
University of Liverpool, Liverpool L69 7ZL, UK  
◇National Institute of Chemical Physics and Biophysics,  
Rävala 10, 10143 Tallinn, Estonia*

<sup>♡</sup>*Department of Physics, University of Ioannina, GR45110 Ioannina, Greece*

## Abstract

The dark matter issue is among the most perplexing in contemporary physics. The problem is more enigmatic due to the wide range of possible solutions, ranging from the ultra-light to the super-massive. String theory gives rise to plausible dark matter candidates due to the breaking of the non-Abelian Grand Unified Theory (GUT) symmetries by Wilson lines. The physical spectrum then contains states that do not satisfy the quantisation conditions of the unbroken GUT symmetry. Given that the Standard Model states are identified with broken GUT representations, and provided that any ensuing symmetry breakings are induced by components of GUT states, leaves a remnant discrete symmetry that forbid the decay of the Wilsonian states. A class of such states are obtained in a heterotic-string derived  $Z'$  model. The model exploits the spinor-vector duality symmetry, observed in the fermionic  $Z_2 \times Z_2$  heterotic-string orbifolds, to generate a  $Z' \in E_6$  symmetry that may remain unbroken down to low energies. The  $E_6$  symmetry is broken at the string level with discrete Wilson lines. The Wilsonian dark matter candidates in the string derived model are  $SO(10)$ , and hence Standard Model, singlets and possess non- $E_6$   $U(1)_{Z'}$  charges. Depending on the  $U(1)_{Z'}$  breaking scale and the reheating temperature they give rise to different scenarios for the relic abundance, and in accordance with the cosmological constraints.

# 1 Introduction

The Standard Model provides viable parameterisation of all experimental data at the subatomic scale. Alas, the Standard Model, and point quantum field theories in general, is not compatible with the gravitational interaction that accounts for observations at the celestial, galactic and cosmological scales. Furthermore, the Standard Model contains only a fraction of the stable matter required to explain the data at the galactic and cosmological scales.

String theory provides a viable framework for perturbative quantum gravity, and gives rise to the gauge and matter states that form the core of the Standard Model. Furthermore, these ingredients arise in string theory due to its internal consistency. String theory, therefore, provides a consistent framework to develop a phenomenological approach to quantum gravity. The phenomenological heterotic-string models constructed in the free fermionic formulation are among the most realistic string models constructed to date. These models are obtained in the vicinity of the self-dual point under  $T$ -duality, providing plausible symmetry arguments to explain their viability, and correspond to  $Z_2 \times Z_2$  toroidal orbifolds, which are among the most symmetric and simplest string compactifications.

The dark matter conundrum is one of the most perplexing puzzles in contemporary observational data. The problem stems from the plethora of possible solutions and the lack of a clear preference for one or the other. Indeed the range of masses for potential candidates extend from  $10^{59}\text{GeV}$  in the form of MACHOS [1] to  $10^{-31}\text{GeV}$  in the form of ultra-light bosons [2]. It is prudent therefore to seek guidance from string theory. In particular, it is sensible to search for potential candidates in phenomenological string constructions.

String models contain in them the favoured dark matter candidates in the form of stable supersymmetric particles and of axion field, as well as other dark matter candidates [3–5]. However, stable supersymmetric dark matter requires reliance on a global symmetry, which ordinarily would be violated in string models [6], whereas recent observational data seem to disfavour a wide range of axion-like candidates [7]. Alternative dark matter candidates in string vacua exist in the form of hidden sector glueball dark matter [3], and Wilsonian dark matter candidates [4]. The latter category arises in string models due to the breaking of the non-Abelian GUT (Grand Unified Theory) gauge symmetries by Wilson lines. The physical spectrum in these string models contains states that do not satisfy the charge quantisation of the unbroken GUT gauge group [8]. Specifically, such states carry fractional charge with respect to some of the unbroken  $U(1)$  generators of the original GUT symmetry. Some of these states may carry fractional electric charge, whereas others may carry standard charges under the Standard Model gauge group, but carry fractional charge with respect to an unbroken  $U(1)_{Z'}$  gauge symmetry. States that carry fractional electric charge are stable by virtue of electric charge conservation. States that carry fractional charge under an unbroken  $U(1)_{Z'}$  symmetry may be stable, depending on

the charges of the states that are used to break the  $U(1)_{Z'}$  symmetry. Breaking  $U(1)_{Z'}$  with Higgs states that carry the standard GUT charges under  $U(1)_{Z'}$  results in a local discrete symmetry that forbids their decay to Standard Model states [4, 9]. Such states may therefore be stable and be viable dark matter candidates. We dub such states as Wilsonian matter states due to the fact that they arise from the breaking of non-Abelian GUT symmetries by Wilson lines.

The possibility of Wilsonian matter states forming the dark matter was studied in ref. [4] for a variety of possible states, including fractionally charged states, strongly interacting states and Standard Model singlet states. The least constrained possibility takes into account the states that carry standard Standard Model charges, but carry fractional charge under a  $U(1)_{Z'}$  gauge symmetry. The Wilsonian states investigated in ref. [4] arise from the symmetry breaking pattern  $SO(10) \rightarrow SU(3) \times SU(2) \times U(1)^2$ . However, the string derived models that utilise this symmetry breaking pattern do not contain the required Higgs states with standard GUT charges to break the  $U(1)_{Z'}$  gauge symmetry [10]. Consequently, these models necessarily utilise Higgs states with fractional  $U(1)_{Z'}$  charges to break the  $U(1)_{Z'}$  along supersymmetric flat direction. More specifically, the state which is missing is the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  singlet in the  $\overline{16}$  representation of  $SO(10)$ . A scan of a large space of similar standard-like vacua may reveal the existence of models that do include the required states [11]. However, barring these new constructions, the string derived model that we discuss in this paper provides the first concrete example that realises the Wilsonian Standard Model singlet dark matter scenario.

The models under consideration are heterotic-string derived models that admit the symmetry breaking pattern  $E_6 \rightarrow SO(10) \times U(1)_\zeta$ , with anomaly free  $U(1)_\zeta$ , in which case  $U(1)_\zeta$  can form part of a low scale  $U(1)_{Z'}$  combination. This is not the case in most of the phenomenological heterotic-string derived models, in which  $U(1)_\zeta$  is anomalous as a generic consequence of the symmetry breaking pattern  $E_6 \rightarrow SO(10) \times U(1)_\zeta$ . The construction of the heterotic-string derived model in ref. [12] utilises the spinor-vector duality that was observed in fermionic  $Z_2 \times Z_2$  orbifolds [13, 14]. The duality operates under the exchange of the total number of  $(16 \oplus \overline{16})$  spinorial  $SO(10)$  representations with the total number of vectorial 10 representations. The models that admit an anomaly free  $U(1)_\zeta$  gauge symmetry are self-dual under the spinor-vector duality. A particular class of models that are self-dual under the spinor-vector duality map are models in which the  $SO(10) \times U(1)_\zeta$  symmetry is enhanced to  $E_6$ . In these models  $U(1)_\zeta$  is anomaly free by virtue of its embedding in  $E_6$ . The total number  $(16 \oplus \overline{16})$  is equal to the total number of 10 representations due to the fact that the 27 and  $\overline{27}$  representations of  $E_6$  contain  $16 + 10$  and  $\overline{16} + 10$ , respectively. Hence,  $E_6$  are self-dual under the exchange of the total number of  $(16 \oplus \overline{16})$  and the total number of 10 representations. However, there exist also self-dual models in which the  $SO(10) \times U(1)_\zeta$  gauge symmetry is not enhanced to  $E_6$ . This is possible if the spinorial and vectorial states are obtained from different fixed points of the  $Z_2 \times Z_2$  orbifold.

The string derived  $Z'$  model of ref. [12] is constructed by first selecting a spinor–vector self–dual model at the  $SO(10)$  level and subsequently breaking the  $SO(10)$  symmetry to the Pati–Salam subgroup. The chiral spectrum of the resulting Pati–Salam string model respects the self–duality under the spinor–vector duality. Effectively, the result is that the chiral spectrum forms complete  $E_6$  multiplets and consequently  $U(1)_\zeta$  is anomaly free.

An unexpected result that was obtained in ref. [12] is with respect to the type of exotic states that appear in the model. Using the trawling algorithm developed for the classification of free fermionic  $Z_2 \times Z_2$  orbifolds [15–19], we fish out a model in which all the exotic fractionally charged states are projected out from the massless spectrum, and appear only as massive states [17]. Such models are dubbed exophobic string models. Therefore, the model does not contain any massless states with fractional charges with respect to the  $SO(10)$  subgroup. However, the model contains exotic states with respect to the  $E_6$  subgroup, *i.e.* the model contains states that are  $SO(10)$  singlets and carry fractional non– $E_6$  charge under  $U(1)_\zeta$ . It is noted that as the gauge symmetries are realised in this model, as level one Kac–Moody algebras the aforementioned exotic charges cannot arise from higher order  $E_6$  representations. Furthermore, the model does contain the required standard  $E_6$  states to break  $U(1)_\zeta$  along flat directions. The model of ref [12] therefore, and for the first time, precisely realises the Wilsonian Standard Model singlet dark matter scenario alluded to in ref. [4].

Our paper is organised as follows: in section 2 we discuss and classify the type of exotic states that arise in the phenomenological fermionic  $Z_2 \times Z_2$  models. We discuss the structure of the models and their construction. In section 3 we elaborate on the exotic  $E_6$  states that are obtained in the  $Z'$  model of ref. [12]. In section 4 we investigate the exotic  $E_6$  states as dark matter candidates, taking into account low and high scale  $U(1)_{Z'}$  breaking as well as scenarios with and without inflation. Section 5 concludes our paper.

## 2 Wilsonian states

The class of string models under consideration are constructed in the free fermionic formulation [20]. The four dimensional heterotic–string in the light–cone gauge requires 20 left–moving, and 44 right–moving, real fermions propagating on the string worldsheet. The sixty–four worldsheet fermions are typically denoted by  $\{\psi^{1,2}, (\chi, y, \omega)^{1,\dots,6}, (\bar{y}, \bar{\omega})^{1,\dots,6}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8}\}$ , where 32 of the right–moving real fermions are grouped into 16 complex fermions that produce the Cartan generators of a rank 16 gauge group. Here  $\bar{\psi}^{1,\dots,5}$  are the Cartan generators of the  $SO(10)$  GUT group and  $\bar{\phi}^{1,\dots,8}$  are the Cartan generators of the rank eight hidden sector gauge group. The three complex worldsheet fermions  $\bar{\eta}^{1,2,3}$  generate three Abelian currents,  $U(1)_{1,2,3}$ , in the Cartan subalgebra of the four dimensional gauge group with  $U(1)_\zeta$

being their linear combination

$$U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3 . \quad (2.1)$$

The worldsheet fermions pick up a phase under parallel transport around one of the non-contractible loops of the vacuum to vacuum torus amplitude. These phases are encoded in forty-eight dimensional vectors  $(20_{l.r.} + 12_{r.r.} + 32_{r.c.})$ ,

$$v = \{v(f_1), \dots, v(f_{20}) | v(\bar{f}_1), \dots, v(\bar{f}_{48})\}.$$

Invariance under modular transformations of the one-loop vacuum to vacuum amplitude leads to a set of constraints on the phase assignments. Summation over all the allowed phases, with appropriate phases to render the sum modular invariant, generates the partition function. The string vacua in the free fermionic formulation are obtained by specifying a set of boundary condition basis vectors,  $B = \{v_1, v_2, v_3, \dots\}$ , and the one-loop summation phases in the partition function  $c_{[v_j]}^{[v_i]}$ . The basis set spans a space  $\Xi$ , which consists of all possible linear combinations of the basis vectors  $\Xi = \sum_k n_k v_k$ , where  $n_k = 0, \dots, N_{v_k} - 1$ , and  $N_{v_k}$  denote the order of each of the basis vectors. The physical states in the Hilbert space of a given sector  $\xi \in \Xi$  are obtained by acting on the vacuum with fermionic and bosonic oscillators and by imposing the Generalised GSO (GGSO) projections. The  $U(1)$  charges with respect to the Cartan generators of the four dimensional gauge group are given by

$$Q(f) = \frac{1}{2}\xi(f) + F_\xi(f),$$

where  $\xi(f)$  is the boundary condition of the complex worldsheet fermion  $f$  in the sector  $\xi$ , and  $F_\xi(f)$  is a fermion number operator [20]. The phenomenological properties of the models are extracted by calculating tree-level and higher order terms in the superpotential and by analysing its flat directions. It is important to note that the free fermionic models correspond to toroidal  $Z_2 \times Z_2$  orbifolds at special points in the moduli space [21]. Moduli deformations of the six dimensional internal torus are incorporated in the fermionic construction in terms of worldsheet Thirring interactions that are consistent with the transformation properties of the worldsheet fermions.

Early examples of quasi-realistic free fermionic models corresponded to the so-called NAHE-based models. The first set of five basis vectors, dubbed the NAHE-set [22], is common in all these phenomenological models, and the models vary by the addition of three or four basis vectors beyond the NAHE-set. Three generation models with  $SU(5) \times U(1)$  (FSU5) [23];  $SO(6) \times SO(4)$  (SO64) [24];  $SU(3) \times SU(2) \times U(1)^2$  (SLM) [10]; and  $SU(3) \times U(1) \times SU(2)^2$  (LRS) [25]  $SO(10)$  subgroup were obtained, whereas the case with  $SU(4) \times SU(2) \times U(1)$  was shown not to produce viable models [26]. In more recent years systematic methods were developed for the classification of large spaces of fermionic  $Z_2 \times Z_2$  heterotic-string vacua. The classification methodology uses an appropriate fixed set of boundary condition basis vectors and the space

of vacua is spanned by varying the GGSO projection coefficients. In this manner models with unbroken  $SO(10)$  gauge group were classified [16], which led to the observation of the spinor–vector duality [13], as well as models with  $SO(6)$  [17] and  $F_{4/24}$  [18]  $SO(10)$  subgroups. Classification of SLM and LRS models is currently underway and will be reported in future publications.

The construction of free fermionic models, in either the older trial and error method, or the more recent systematic classification method, can be viewed in two stages. The first part consist of the basis vectors that preserve the  $SO(10)$  symmetry. The construction at this stage produces vacua with  $(2, 0)$  worldsheet supersymmetry,  $N = 1$  spacetime supersymmetry, and a number of spinorial and vectorial representations of  $SO(10)$ . The second part consist of the inclusion of the basis vectors that break the  $SO(10)$  symmetry to a subgroup.

Correspondingly, the sectors in a free fermionic heterotic-string model can be divided into those that preserve the  $SO(10)$  symmetry and those that do not. Physical states that arise from sectors that preserve the  $SO(10)$  symmetry correspond to states that may be identified with Standard Model states, or are Standard Model singlets. Sectors that break the  $SO(10)$  symmetry produce exotic states, *i.e.* they produce states that carry fractional charge under  $U(1)_{\text{e.m.}}$  or under  $U(1)_{Z'} \in SO(10)$ . The exotic states can be further classified according to the  $SO(10)$  symmetry breaking pattern in the sector from which they arise.

The Cartan subalgebra of the observable gauge group in the free fermionic models is generated by the complex worldsheet fermions  $\{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\}$ , with  $\bar{\psi}^{1,\dots,5}$  producing those of  $SO(10)$  and its subgroups and  $\bar{\eta}^{1,2,3}$  producing three  $U(1)$  currents. The  $SO(10)$  symmetry is broken to one of its subgroups by the following assignments:

$$1. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \left\{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}\right\} \Rightarrow SU(5) \times U(1), \quad (2.2)$$

$$2. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \{1 \ 1 \ 1 \ 0 \ 0\} \Rightarrow SO(6) \times SO(4). \quad (2.3)$$

To break the  $SO(10)$  symmetry to  $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$  [10] both steps, 1 and 2, are used, in two separate basis vectors<sup>1</sup>. The breaking pattern  $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  [25] is obtained with:

$$3. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \left\{\frac{1}{2} \frac{1}{2} \frac{1}{2} 0 0\right\} \Rightarrow SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R, \quad (2.4)$$

and the breaking pattern  $SO(10) \rightarrow SU(4)_C \times SU(2)_L \times U(1)_R$  [26] results from:

$$4. \quad b\{\bar{\psi}_{\frac{1}{2}}^{1\dots 5}\} = \left\{0 0 0 \frac{1}{2} \frac{1}{2}\right\} \Rightarrow SU(4)_C \times SU(2)_L \times U(1)_R. \quad (2.5)$$

It was shown that the breaking pattern (2.5) does not produce viable models [26].

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<sup>1</sup> $U(1)_C = \frac{3}{2}U(1)_{B-L}; U(1)_L = 2U(1)_{T_{3R}}.$

The states in the free fermionic models that carry exotic charges with respect to the Abelian generators of the  $SO(10)$  subgroups can be classified according to the  $SO(10)$  symmetry breaking pattern in the sectors from which they arise. A basis vector combination that produces exotic states contains in it the  $SO(10)$  breaking basis vectors. We focus here on the case of the standard-like models that contain both of the assignments shown in (2.2) and in (2.3) and therefore contain the exotic states that arise in the FSU5 and the SO64 models, as well as those that arise solely in the SLM models. In the following we adapt the notation

$$[(SU(3)_C \times U(1)_C); (SU(2)_L \times U(1)_L)]_{(Q_Y, Q_{Z'}, Q_{\text{e.m.}})}, \quad (2.6)$$

to denote the charges of the states arising in the exotic sectors. Here  $U(1)_C$  and  $U(1)_L$  are defined in terms of the worldsheet charges by

$$Q_C = Q(\bar{\psi}^1) + Q(\bar{\psi}^2) + Q(\bar{\psi}^3) \text{ and } Q_L = Q(\bar{\psi}^4) + Q(\bar{\psi}^5). \quad (2.7)$$

The FSU5  $U(1)$  combinations are given by

$$U(1)_5 = \frac{1}{3}U(1)_C - \frac{1}{2}U(1)_L \in SU(5) \quad (2.8)$$

$$U(1)_{\bar{5}} = U(1)_C + U(1)_L \notin SU(5) \quad (2.9)$$

whereas the weak hypercharge and  $Z'$  charges in eq. (2.6) are given by

$$U(1)_Y = \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L = \frac{2}{5}U(1)_5 - \frac{1}{5}U(1)_{\bar{5}} \quad (2.10)$$

$$U(1)_{Z'} = U(1)_C - U(1)_L = \frac{12}{5}U(1)_5 + \frac{1}{5}U(1)_{\bar{5}}. \quad (2.11)$$

$U(1)_C$  and  $U(1)_L$  are similarly defined in the SO64 models. The electromagnetic charge is given by

$$U(1)_{\text{e.m.}} = T_{3_L} + U(1)_Y. \quad (2.12)$$

Using the notation in eq. (2.6) the Standard Model matter states carry the charges

$$e_L^c \equiv [(1, \frac{3}{2}); (1, 1)]_{(1, 1/2, 1)}; \quad (2.13)$$

$$u_L^c \equiv [(\bar{3}, -\frac{1}{2}); (1, -1)]_{(-2/3, 1/2, -2/3)}; \quad (2.14)$$

$$Q \equiv [(3, \frac{1}{2}); (2, 0)]_{(1/6, 1/2, (2/3, -1/3))}; \quad (2.15)$$

$$N_L^c \equiv [(1, \frac{3}{2}); (1, -1)]_{(0, 5/2, 0)}; \quad (2.16)$$

$$d_L^c \equiv [(\bar{3}, -\frac{1}{2}); (1, 1)]_{(1/3, -3/2, 1/3)}; \quad (2.17)$$

$$L \equiv [(1, -\frac{3}{2}); (2, 0)]_{(-1/2, -3/2, (0, 1))}, \quad (2.18)$$

and arise from spinorial 16 representations of  $SO(10)$ , *i.e.* they arise from sectors that preserve the  $SO(10)$  gauge symmetry. Similarly, the light Higgs electroweak doublets are obtained from  $SO(10)$  vectorial representations, and arise in sectors that preserve the  $SO(10)$  symmetry. By contrast the exotic states arise in sectors that break the  $SO(10)$  symmetry and can be classified according to the  $SO(10)$  symmetry breaking pattern in each sector. Sectors that break the  $SO(10)$  symmetry to the FSU5 subgroup contain in them the assignment in eq. (2.2) and produce the states

$$[(3, -\frac{1}{4}); (1, \frac{1}{2})]_{(1/6, -3/4, 1/6)} \quad ; \quad [(\bar{3}, \frac{1}{4}); (1, -\frac{1}{2})]_{(-1/6, 3/4, -1/6)}, \quad (2.19)$$

$$[(1, \frac{3}{4}); (2, -\frac{1}{2})]_{(0, 5/4, \pm 1/2)} \quad ; \quad [(1, -\frac{3}{4}); (2, \frac{1}{2})]_{(0, -5/4, \pm 1/2)}, \quad (2.20)$$

$$[(1, \frac{3}{4}); (1, \frac{1}{2})]_{(1/2, 1/4, 1/2)} \quad ; \quad [(1, -\frac{3}{4}); (1, -\frac{1}{2})]_{(-1/2, -1/4, -1/2)}. \quad (2.21)$$

Sectors that break the  $SO(10)$  symmetry to the SO64 subgroup contain in them the assignment in eq. (2.3) and produce the states

$$[(3, \frac{1}{2}); (1, 0)]_{(1/6, 1/2, 1/6)} \quad ; \quad [(\bar{3}, -\frac{1}{2}); (1, 0)]_{(-1/6, -1/2, -1/6)}, \quad (2.22)$$

$$[(1, \frac{3}{2}); (1, 0)]_{(1/2, 3/2, 1/2)} \quad ; \quad [(1, -\frac{3}{2}); (1, 0)]_{(-1/2, -3/2, -1/2)}, \quad (2.23)$$

$$[(1, 0); (1, 1)]_{(1/2, -1, 1/2)} \quad ; \quad [(1, 0); (1, -1)]_{(-1/2, 1, -1/2)}, \quad (2.24)$$

$$[(1, 0); (2, 0)]_{(0, 0, \pm 1/2)} \quad . \quad (2.25)$$

Sectors that break the  $SO(10)$  symmetry to the SLM subgroup contain a linear combination of both assignments in eq.(2.2) and eq. (2.3). These sectors produce states that carry standard GUT charges with respect to the Standard Model gauge group but carry fractional non-GUT charges with respect to the  $U(1)_{Z'}$  combination in eq. (2.11):

$$[(3, -\frac{1}{4}); (1, -\frac{1}{2})]_{(-1/3, 1/4, -1/3)} \quad ; \quad [(\bar{3}, \frac{1}{4}); (1, \frac{1}{2})]_{(1/3, -1/4, 1/3)}, \quad (2.26)$$

$$[(1, \frac{3}{4}); (2, -\frac{1}{2})]_{(1/2, 3/4, (1,0))} \quad ; \quad [(1, -\frac{3}{4}); (2, \frac{1}{2})]_{(-1/2, -3/4, (0,-1))}, \quad (2.27)$$

$$[(1, \frac{3}{4}); (1, -\frac{1}{2})]_{(0, 5/4, 0)} \quad ; \quad [(1, -\frac{3}{4}); (1, -\frac{1}{2})]_{(0, -5/4, 0)}. \quad (2.28)$$

The exotic states appearing in eqs. (2.26, 2.27, 2.28) may therefore produce viable dark matter candidates. This would be the case if the heavy Higgs states that break  $U(1)_{Z'}$  carry the standard GUT charges with respect to  $U(1)_{Z'}$ . In that case a remnant discrete symmetry forbids the formation of unsuppressed terms that can lead to decay of the exotic states to the Standard Model states [4]. We remark that all nonrenormalisable gauge invariant operators that may be formed are suppressed



by at least one power of  $M_{\text{String}}$ . They are therefore sufficiently small and cannot lead to rapid decay of the Wilsonian dark matter states [4]. In the FSU5 and SO64 heavy Higgs states necessarily arise from GUT representation in order to break the remaining non-Abelian symmetry. However, in the SLM models this need not be the case. The remnant unbroken symmetry  $U(1)_{Z'}$  of eq. (2.11) can be broken by heavy Higgs states with the standard GUT charges of eq. (2.16) and its conjugate  $\bar{N}_L^c$ , or by using the exotic states and charges in eq. (2.28). In the SLM models of [10], a state with the quantum numbers of  $\bar{N}_L^c$  does not appear in the massless spectrum. Consequently, in these models, breaking  $U(1)_{Z'}$  and preserving supersymmetry at a high scale forces the exotic states in eq. (2.28) to get a non-trivial VEV at the high scale. Alternatively, we may contemplate that either the  $U(1)_{Z'}$  symmetry is broken at the low scale, or that supersymmetry is broken at the high scale. Suppression of left-handed neutrino masses by the seesaw mechanism disfavors the first possibility. Breaking supersymmetry at the high scale also introduces a plethora of new questions that we do not consider in this paper. The upshot is that the Wilsonian singlet dark matter scenario of [4], with the type singlets in eq. (2.28), is not realised in the existing SLM heterotic-string free fermionic models.

### 3 $E_6$ Wilsonian states in $Z'$ string model

We next turn to discuss the Wilsonian matter states in the string derived model of ref. [12]. The  $SO(10)$  symmetry is broken in this model to the Pati-Salam subgroup. The chiral spectrum of the model forms complete  $E_6$  multiplets. Consequently, the  $U(1)_\zeta$  combination, which possesses the embedding  $SO(10) \times U(1)_\zeta \in E_6$  is anomaly free. The complete massless spectrum of the model, and its charges under the four dimensional gauge group, is given in ref. [12]. Here a glossary of the states in the model and the charges under the  $SU(4) \times SO(4) \times U(1)_\zeta$  are shown in tables 1 and 2. The heterotic-string model in ref. [12] is an exophobic Pati-Salam model. The type of massless exotic states that can appear in this model are those in eqs. (2.22, 2.25, 2.23, 2.24). However, none of these states appear in the massless spectrum. In fact, none of the exotic states discussed in section 2 appear in this model.

The model contains, however, a new type of exotic states. These exotic states carry standard  $SO(10)$  charges and are in fact  $SO(10)$  singlets. They are exotic with respect to  $U(1)_\zeta$  as they carry 1/2 of the charge of the standard  $SO(10)$  singlets in the 27 and  $\bar{27}$  representations of  $E_6$ . Inspection of table 1 shows that the exotic states of this type are  $\{\phi_{1,2}, \bar{\phi}_{1,2}\}$ , which are also singlets of the rank 8 hidden sector gauge group. The  $SO(10)$  singlet states  $H^+$  and  $H^-$  in table 2 carry similar  $U(1)_\zeta$  charges and transform under the hidden sector gauge group. These exotic states arise in the string models due to the breaking of the  $E_6$  symmetry by Wilson lines. However, as the Wilson line is realised in the free fermionic construction in terms of a GGSO phase, its precise identification is not a simple exercise. Its imprint is revealed due

Symbol	Fields in [12]	$SU(4) \times SU(2)_L \times SU(2)_R$	$U(1)_\zeta$
$F_L$	$F_{1L}, F_{2L}, F_{3L}$	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$+\frac{1}{2}$
$F_R$	$F_{1R}$	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$	$-\frac{1}{2}$
$\bar{F}_R$	$\bar{F}_{1R}, \bar{F}_{2R}, \bar{F}_{3R}, \bar{F}_{4R}$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	$+\frac{1}{2}$
$h$	$h_1, h_2, h_3$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$-1$
$\Delta$	$D_1, \dots, D_7$	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$-1$
$\bar{\Delta}$	$\bar{D}_1, \bar{D}_2, \bar{D}_3, \bar{D}_6$	$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$+1$
$S$	$\Phi_{12}, \Phi_{13}, \Phi_{23}, \chi_1^+, \chi_2^+, \chi_3^+, \chi_5^+$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$+2$
$\bar{S}$	$\bar{\Phi}_{12}, \bar{\Phi}_{13}, \bar{\Phi}_{23}, \bar{\chi}_4^+$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$-2$
$\phi$	$\phi_1, \phi_2$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$+1$
$\bar{\phi}$	$\bar{\phi}_1, \bar{\phi}_2$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$-1$
$\zeta$	$\Phi_{12}^-, \Phi_{13}^-, \Phi_{23}^-, \bar{\Phi}_{12}^-, \bar{\Phi}_{13}^-, \bar{\Phi}_{23}^-$ $\chi_1^-, \chi_2^-, \chi_3^-, \bar{\chi}_4^-, \chi_5^-$ $\zeta_i, \bar{\zeta}_i, i = 1, \dots, 9$ $\Phi_i, i = 1, \dots, 6$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$	$0$

Table 1: Observable sector field notation and associated states in [12].

Symbol	Fields in [12]	$SU(2)^4 \times SO(8)$	$U(1)_\zeta$
$H^+$	$H_{12}^3$	$(\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1})$	$+1$
	$H_{34}^2$	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})$	$+1$
$H^-$	$H_{12}^2$	$(\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1})$	$-1$
	$H_{34}^3$	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})$	$-1$
$H$	$H_{12}^1$	$(\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{1})$	$0$
	$H_{13}^i, i = 1, 2, 3$	$(\mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1})$	$0$
	$H_{14}^i, i = 1, 2, 3$	$(\mathbf{2}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{1})$	$0$
	$H_{23}^1$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1})$	$0$
	$H_{24}^1$	$(\mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1})$	$0$
	$H_{34}^i, i = 1, 4, 5$	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1})$	$0$
$Z$	$Z_i, i = 1, \dots,$	$(\mathbf{1}, \mathbf{1}, \mathbf{8})$	$0$

Table 2: Hidden sector field notation and associated states in [12].

to the exotic charges, which will not have been generated otherwise. Furthermore, we note from table 1 that the string model does contain the  $SO(10)$  singlet states  $S$  and  $\bar{S}$ , with standard  $E_6$  charges to break  $U(1)_\zeta$  along flat directions. Therefore, this model can realise the Wilsonian singlet dark matter scenario articulated in ref. [4]. In the next section we turn to examine this question.

## 4 $SO(10)$ singlet Wilsonian dark matter

In this section we examine several scenarios in which the Wilsonian matter states can account for the dark matter without overclosing the universe. Our analysis here is primarily qualitative and more detailed numerical analysis will be reported in future work. As discussed in the previous sections, the main feature of the Wilsonian matter states is the existence of an intrinsic stringy mechanism that produces stable matter states. In ref. [4] a similar analysis was performed for states that carry Standard Model, or  $SO(10)$  charges. The novelty here is that the Wilsonian states arise as  $SO(10)$  singlets and interact with the Standard Model states only via the  $U(1)_{Z'}$  couplings. We note that contrary to other dark matter candidates in the literature, whose stability relies on the existence of global gauge or discrete symmetry, the stability of the Wilsonian states arises from the assumption that the Wilsonian states themselves do not receive a vacuum expectation value. As seen from table 1, the heterotic-string model of ref. [12] allows this assumption to be made because it contains the standard  $E_6$  charged states  $F_R$ ,  $\bar{F}_R$  and  $S$ ,  $\bar{S}$  to break the gauge symmetry along flat directions.

We next comment on the allowed values of the gauge and Yukawa couplings of the Wilsonian states. The entire cubic level superpotential was presented in ref. [12]. All the couplings in heterotic-string model are given in terms of the unified gauge coupling and the VEVs of some moduli fields. The relevant parameter for the calculation of the relic abundance is the  $Z'$  gauge coupling, subject to the assumptions on the  $Z'$  breaking scale, the mass scale of the dark matter states and the reheating temperature. These three scales are taken as input parameters and the constraints on the relic abundance are obtained subject to some initial assumptions (i.e. thermal or non-thermal relic) by solving the Boltzmann equation. We consider both high and low scale  $U(1)_{Z'}$  breaking. In the low scale breaking scenario the  $Z'$  mass scale can be generated dynamically, in which case the relevant parameters are cubic level coupling in the string derived superpotential, which are all given in terms of the unified gauge coupling. The numerical value of the gauge coupling at the unification scale is constrained by compatibility with the gauge coupling data at the electroweak scale. The  $Z'$  is constrained by the LHC experiments to be heavier than a few TeVs and we may therefore assume that it is heavier than  $\simeq 4\text{TeV}$ . In this case its mixing with the Standard Model  $Z$ -boson is small and does not affect the analysis. A detailed numerical analysis is beyond the scope of this paper, and will be reported in future publications.

The low energy spectrum of the string model consists of the states in table 3, where we also allow for the possibility of completely neutral states that may correspond to light hidden sector states.

The trilinear superpotential embedding the supersymmetric  $Z'$  model and the

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{Z'}$
$Q_L^i$	3	2	$+\frac{1}{6}$	$-\frac{2}{5}$
$u_L^i$	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{5}$
$d_L^i$	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{5}$
$e_L^i$	1	1	+1	$-\frac{2}{5}$
$L_L^i$	1	2	$-\frac{1}{2}$	$-\frac{4}{5}$
$D^i$	3	1	$-\frac{1}{3}$	$+\frac{4}{5}$
$\bar{D}^i$	$\bar{3}$	1	$+\frac{1}{3}$	$+\frac{6}{5}$
$H^i$	1	2	$-\frac{1}{2}$	$+\frac{6}{5}$
$\bar{H}^i$	1	2	$+\frac{1}{2}$	$+\frac{4}{5}$
$S^i$	1	1	0	-2
$\mathcal{D}$	3	1	$-\frac{1}{3}$	$+\frac{4}{5}$
$\bar{\mathcal{D}}$	$\bar{3}$	1	$+\frac{1}{3}$	$-\frac{4}{5}$
$\phi$	1	1	0	-1
$\bar{\phi}$	1	1	0	+1
$\zeta^i$	1	1	0	0

Table 3: *Spectrum and  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'}$  quantum numbers, with  $i = 1, 2, 3$  for the three light generations. The charges are displayed in the normalisation used in free fermionic heterotic-string models.*

relevant interactions of the  $\phi$  and  $\bar{\phi}$  is

$$\begin{aligned}
\mathcal{W} = & Y^u_{i,j,k} \bar{u}_R^i Q_L^j \bar{H}^k - Y^d_{i,j,k} \bar{d}_R^i Q_L^j H^k - Y^e_{i,j,k} \bar{e}_R^i L_L^j H^k \\
& + \lambda_{i,j,k} S^i H^j \bar{H}^k + \kappa_{i,j,k} S^i \bar{D}^j D^k + \lambda^s_i S^i \bar{\phi} \phi .
\end{aligned} \tag{4.1}$$

As seen from eq. (4.1) there are no terms that allow for the  $\phi, \bar{\phi}$  states to decay to lighter states at leading order. Breaking the  $U(1)_{Z'}$  symmetry with the VEV of  $S^i$  leaves a remnant discrete symmetry [4, 9] which forbids their decay at any order in the superpotential. A potential mass term for  $\bar{\phi}$  arises in the trilinear superpotential, eq. (4.1). Additional mass terms, that are invariant under all gauge and discrete symmetries, may be generated from higher order terms. Therefore, the Wilsonian matter states, namely the fermionic component arising from the  $\phi, \bar{\phi}$  states, in the string derived  $Z'$  model are heavy and stable. Their mass density may overclose the universe if they are over abundant. We refer to these states as Wilsonian singlet dark matter, or  $W_s$  for short.

The Wilsonian singlet interacts with the non-exotic states in table 3 only via the  $Z'$  gauge charges. Its number density can change only by annihilations via the diagrams in figures 1 and 2 into fermions and their superpartners, and, depending on the  $U(1)_{Z'}$  symmetry breaking scale, into the gauge bosons and their superpartners, as in figures 3 and 4.

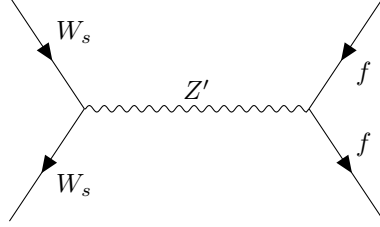


Figure 1:  $W_s \bar{W}_s \longrightarrow f \bar{f}$  decay.

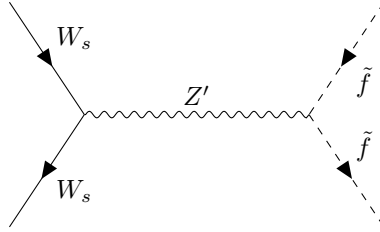


Figure 2:  $W_s \bar{W}_s \longrightarrow \tilde{f} \tilde{f}^*$  decay.

The annihilation into fermion and sfermion in figures 1 and 2 leads to the two contributions

$$\sigma_{W_s W_s \rightarrow f \bar{f}} = \frac{4\pi}{3} \frac{N_{Z'} \sqrt{S} (2M_{W_s}^2 + S)}{\sqrt{S - 4M_{W_s}^2} (M_{Z'}^2 - S)^2} \quad (4.2)$$

and

$$\sigma_{W_s W_s \rightarrow \tilde{f} \tilde{f}^*} = \frac{\pi}{3} \frac{N_{Z'} \sqrt{S} (2M_{W_s}^2 + S)}{\sqrt{S - 4M_{W_s}^2} (M_{Z'}^2 - S)^2} \quad (4.3)$$

where we defined the parameter

$$N_{Z'} = \frac{g'^4}{16\pi^2} Q_f^2 Q_{W_s}^2 \quad (4.4)$$

with  $Q_i$  and  $g'$  being the  $U(1)_{Z'}$  charge and the corresponding coupling constant, respectively. The  $N_{Z'}$  parameter accounts for the strength of a  $Z'$  exchange. The total cross-section for the annihilation into fermions and sfermions  $W_s \bar{W}_s \rightarrow f \bar{f}, \tilde{f}_L \tilde{f}_L^*, \tilde{f}_R \tilde{f}_R^*$  is now easily reached

$$\sigma_{W_s W_s \rightarrow f \bar{f}, \tilde{f} \tilde{f}} = \sigma_{W_s W_s \rightarrow f \bar{f}} + 2 \sigma_{W_s W_s \rightarrow \tilde{f} \tilde{f}} = \frac{2 \pi N_{Z'} \sqrt{S} (2 M_{W_s}^2 + S)}{\sqrt{S - 4 M_{W_s}^2} (M_{Z'}^2 - S)^2} \quad (4.5)$$

where the contribution of (4.3) has been doubled to account for the separate events  $W_s \bar{W}_s \rightarrow \tilde{f}_L \tilde{f}_L^*$  and  $W_s \bar{W}_s \rightarrow \tilde{f}_R \tilde{f}_R^*$  which have identical cross-sections.

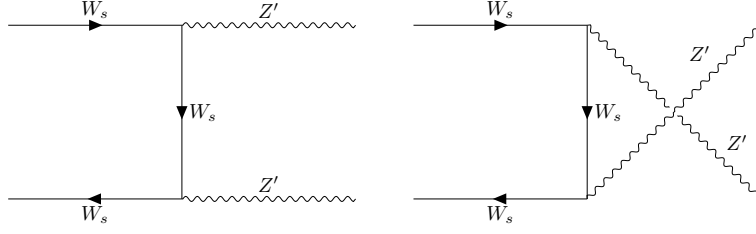


Figure 3:  $W_s \bar{W}_s \rightarrow Z' Z'$  decay.

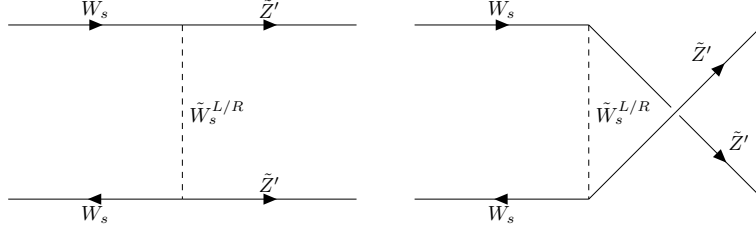


Figure 4:  $W_s \bar{W}_s \rightarrow \tilde{Z}' \tilde{Z}'$  decay.

The computation of the annihilations into vector bosons in figure 3 and into their superpartners in figure 4 leads to the cross-sections

$$\sigma_{W_s W_s \rightarrow Z' Z'} = 4 \pi \tilde{N}_{Z'} \frac{(S^2 + 4 M_{W_s}^2 S - 8 M_{W_s}^4) \ln \left( \frac{S+A}{S-A} \right) - A (S + 4 M_{W_s}^2)}{A^2 S} \quad (4.6)$$

and

$$\sigma_{W_s W_s \rightarrow \tilde{Z}' \tilde{Z}'} = 2 \pi \tilde{N}_{Z'} \frac{A - 2 M_{W_s}^2 \ln \left( \frac{S+A}{S-A} \right)}{A^2} \quad (4.7)$$

where we have defined  $\tilde{N}_{Z'}$  as

$$\tilde{N}_{Z'} = \frac{g'^4}{16 \pi^2} Q_{W_s}^4 \quad (4.8)$$

and the kinematical parameter  $A = \sqrt{S(S - 4M_{W_s}^2)}$ . Since  $W_s$  is heavy and stable, its mass density is constrained by the requirement that it does not overclose the universe. Alternatively, we may extract the regions of parameter space where the non-baryonic dark matter abundance can be explained in terms of the Wilsonian singlet dark matter. After  $U(1)_{Z'}$  symmetry breaking the  $W_s$  interactions are suppressed by  $1/M_{Z'}^2$ , and it can be classified as weakly interacting massive particle. It decouples from the thermal bath when its annihilation rate falls behind the expansion rate of the universe. The annihilation rate of a particle is

$$\Gamma = \langle \sigma_{\text{ann}} |v| \rangle n_{EQ}, \quad (4.9)$$

where the number density at the equilibrium,  $n_{EQ}$ , is given by

$$n_{EQ} = \begin{cases} g_{\text{eff}} \left( \frac{\zeta(3)}{\pi^2} \right) T^3 & \text{relativistic} \\ g_{\text{eff}} \left( \frac{mT}{2\pi} \right)^{3/2} \exp(-M/T) & \text{non-relativistic} \end{cases} \quad (4.10)$$

and  $\zeta(3) = 1.20206$  is the Riemann zeta function of 3, and  $g_{\text{eff}}$  is the effective number of degrees of freedom of the particle. In the expanding universe the evolution equation of the number density is described by the Boltzmann equation [28]. In terms of the number density in a comoving volume  $Y = n/s_e$ , where  $s_e$  is the entropy density, and the dimensionless parameter  $x = M/T$ , the Boltzmann equation takes the form

$$\frac{dY}{dx} = -\lambda x^{-2} (Y^2 - Y_{EQ}^2), \quad (4.11)$$

with  $\lambda$  related to the interaction rate of the particle through

$$\lambda = 0.26 < \sigma_{\text{ann}} |v| > M_{W_s} m_{Pl} \frac{g_{*s}}{\sqrt{g_*}} \quad (4.12)$$

with  $Y_{EQ}$  the density at thermal equilibrium and  $m_{Pl}$  the Planck Mass. In (4.12) the variables  $g_*$  and  $g_{*s}$  count, for different purposes, the effective relativistic degrees of freedom at a given  $T$

$$\begin{aligned} g_* &= \sum_{i=\text{bosons}} g_i \left( \frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left( \frac{T_i}{T} \right)^4, \\ g_{*s} &= \sum_{i=\text{bosons}} g_i \left( \frac{T_i}{T} \right)^3 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left( \frac{T_i}{T} \right)^3, \end{aligned} \quad (4.13)$$

where  $g_i$  are the complete internal degrees of freedom and  $T_i$  the temperature of the given particle  $i$ . The annihilation cross-section  $\sigma_{ann}$  determines the evolution of the relic density via its thermal average,  $\langle \sigma_{ann}|v| \rangle$ , and its computation must be performed to calculate the relic abundance left by  $W_s$ . We can distinguish different scenarios for the Wilsonian singlet dark matter, depending on the  $Z'$  symmetry breaking scale,  $M_{Z'}$ , the mass scale of  $W_s$ ,  $M_{W_s} \equiv M$ , and the reheating temperature,  $T_R$ , following inflation. The  $Z'$  breaking scale can vary from the experimental LHC mass limits, of the order of a few TeV, up to the Planck scale. The constraints on the  $W_s$  mass can vary from being ultra light [2] to being super-massive [4, 29], depending on the  $Z'$  breaking scale, and the reheating temperature. There are several possible dark matter scenarios for the Wilsonian singlet  $W_s$ :

- **$M_{W_s} \gg M_{Z'}$  without inflation.** In this case the Wilsonian singlet is strongly interacting in the early universe and remains in thermal equilibrium until it becomes non-relativistic. The solution of the Boltzmann equation leads to a density value of

$$Y_0 = \frac{3.79}{m_{Pl} T_{dec} \langle \sigma_{ann}|v| \rangle} \left( \frac{\sqrt{g_*}}{g_{*s}} \right). \quad (4.14)$$

In this scenario all the annihilation channels depicted in figures 1, 2, 3, 4 are open. The  $s$ -wave expansion of the Boltzmann equation [28] yields the thermal average of (4.5), (4.6) and (4.7) as:

$$\langle \sigma_{W_s W_s \rightarrow f \bar{f}, \tilde{f} \tilde{\bar{f}}}|v| \rangle = \frac{8\pi N_{Z'}}{3 M_{W_s}^2} + O(v^2) \quad (4.15)$$

and

$$\langle \sigma_{W_s W_s \rightarrow Z' Z'}|v| \rangle + \langle \sigma_{W_s W_s \rightarrow \tilde{Z}' \tilde{Z}'}|v| \rangle = \frac{4\pi \tilde{N}_{Z'}}{M_{W_s}^2} + O(v^2) \quad (4.16)$$

which sums with (4.15) for the total annihilation cross-section of the singlet in the regime  $M_{W_s} \gg M_{Z'}$

$$\langle \sigma_{ann}|v| \rangle = \sigma_0 + O(v^2) = \frac{4\pi}{M_{W_s}^2} \left( \frac{2}{3} N_{Z'} + \tilde{N}_{Z'} \right) + O(v^2). \quad (4.17)$$

To complete the computation of the number density (4.14) the decoupling temperature is inferred by exploiting, from the Boltzmann equation, the condition  $dY/dx \simeq 0$  which results in [28]

$$\frac{T_{dec}}{M_{W_s}} \simeq \left( \ln(0.038 (g/\sqrt{g_*}) m_{Pl} M_{W_s} \sigma_0) - \frac{1}{2} \ln(\ln(0.038 (g/\sqrt{g_*}) m_{Pl} M_{W_s} \sigma_0)) \right)^{-1}.$$



Its easy to show that the decoupling temperature has a very mild dependence over the model dependent factors  $N_{Z'}$  and  $\tilde{N}_{Z'}$ , as well as over realistic values of the variable  $g_*$ . A solid and simple fit is found using the formula [4]

$$T_{dec} = \frac{M_{W_s}}{\ln(m_{Pl}/M_{W_s})}, \quad (4.18)$$

which allows to find the relation for the density  $Y_0$  in eq. (4.14)

$$Y_0 \simeq \frac{0.3 M_{W_s} \ln(m_{Pl}/M_{W_s})}{\sqrt{g_*} m_{Pl} (\frac{2}{3} N_{Z'} + \tilde{N}_{Z'})} \quad (4.19)$$

where the approximation  $g_* \sim g_{*s}$  can be used as the decoupling temperature is sufficiently high in the relevant region of parameter space. Finally, to draw an estimate for the value of  $\Omega h^2$ , we first find the current energy density  $\rho_0$  by multiplying for the entropy  $s_{e0}$ ,  $\rho_0 = s_{e0} Y_0 M$ , and then divide by the critical energy  $\rho_c$  to arrive at the final expression

$$\Omega h^2 = \frac{\rho_0 h^2}{\rho_c} = \frac{2970 M_{W_s} Y_0 \text{ cm}^{-3}}{1.05 \cdot 10^4 \text{ eV cm}^{-3}}. \quad (4.20)$$

Taking  $\Omega h^2 \sim 1$  we obtain an upper bound

$$M_{W_s} < 10^5 \sqrt{\frac{\sqrt{g_*} \mathcal{N}}{\ln(m_{Pl}/M_{W_s})}} \text{ GeV} \quad (4.21)$$

with  $\mathcal{N} = \left(\frac{2}{3} N_{Z'} + \tilde{N}_{Z'}\right)$  determined by the  $U(1)_{Z'}$  couplings in table 3. We note that  $U(1)_{Z'}$  symmetry is restored when the temperature goes above its breaking scale. The extra vector-like matter states, beyond the Standard Model, in table 3, become massive at that scale. Their effect above that scale is incorporated in the analysis by summing over the charges in the numerical factor under the square root in eq. (4.21). By an explicit display of the bound in eq. (4.21) we may estimate the values of  $g'$  and  $M_{W_s}$  that are required, in such a scenario, to avoid an over abundant Wilsonian state relic. The region allowed by (4.21) in figure 5 reveals how, to avoid too light  $Z'$  mass and simultaneously respect the hierarchy  $M_{W_s} \gg M_{Z'}$ , a large, but perturbative,  $g'$  must be adopted. We notice that perturbativity of the  $U(1)_{Z'}$  coupling requires an upper bound of  $\simeq 25\text{TeV}$  on the mass of the Wilsonian dark matter as thermal relic with low scale  $U(1)_{Z'}$  breaking.

- $M_{W_s} \gg M_{Z'}$  **with inflation.** The current relic density could also be generated by an out-of-equilibrium production after reheating if such process takes

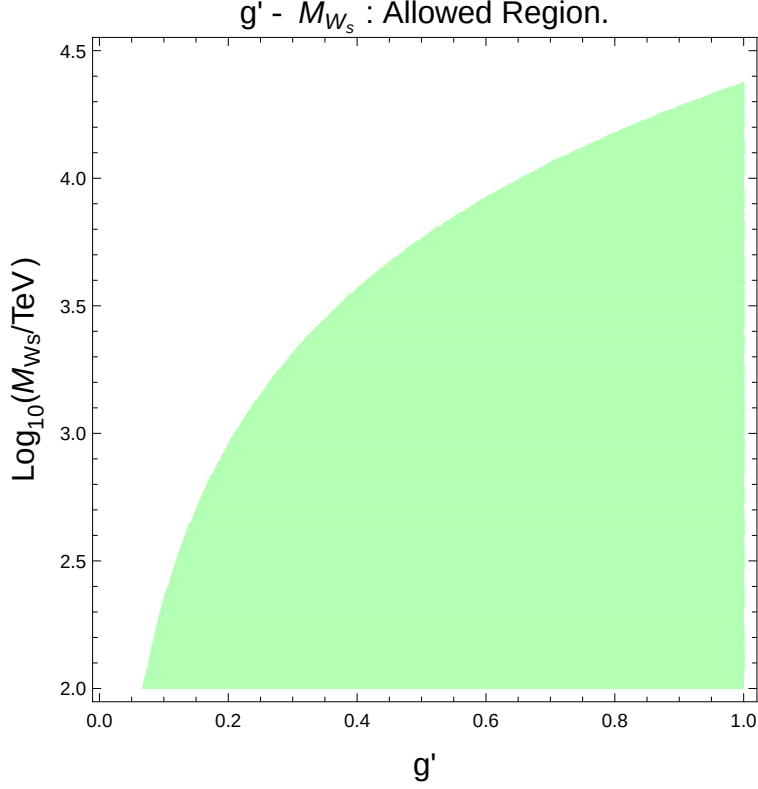


Figure 5: *In green: region in the  $g' - M_{W_s}$  space preventing overabundant Dark Matter in the case  $M_{W_s} \gg M_{Z'}$  without inflation.*

place after singlet decoupling. In this case inflation will dilute the singlet density and, from the Boltzmann equations (4.11), we can study the subsequent evolution with the approximate knowledge of the density derivative,

$$\frac{dY}{dx} = \lambda x^{-2} Y_{EQ}^2, \quad (4.22)$$

with  $Y_{EQ} = 0.145 g_{eff}/g_{*s} x^{3/2} e^{-x}$  (non relativistic case). When  $\lambda$  is independent of  $x$ , eq. (4.22) can be promptly integrated to the present temperature, providing the density produced by the singlet after reheating [4]

$$Y_0 = \frac{\lambda g_{eff}^2}{2} \left( \frac{0.145}{g_*} \right)^2 \left( \frac{M_{W_s}}{T_R} + \frac{1}{2} \right) e^{-2 \frac{M_{W_s}}{T_R}}, \quad (4.23)$$

with  $T_R$  the reheating temperature. Such value, inserted in (4.20) and requiring that it does not exceed the measured cosmological relic density, results in a bound, involving the  $Z'$  and  $W_s$  masses as well as  $T_R$ , whose specific form depends on the process through which the final density is regenerated. As long as the reheating temperature is greater than the  $Z'$  mass, the generation of the

singlets will take place with all the channels investigated above, so that the relevant (thermal) cross-section is (4.17). Using (4.23), the cosmological upper bound over the relic is obtained with the condition

$$M_{W_s} > T_R \left( 26 + \frac{1}{2} \ln \left( \frac{M_{W_s}}{T_R} + \frac{1}{2} \right) + \frac{1}{2} \ln(\mathcal{N}) \right) \quad (4.24)$$

and  $\mathcal{N} = \left( \frac{2}{3} N_{Z'} + \tilde{N}_{Z'} \right)$ .

- **$M_{W_s} \ll M_{Z'}$  without inflation.** In this case  $W_s$  is a WIMP and it can only annihilate into the matter supermultiplets in table 3 via the diagrams in figures 1 and 2, which are suppressed by the heavy  $Z'$  vector boson mass. The interaction between  $W_s$  and other matter states will be kinematically suppressed when  $T$  is below  $M_{Z'}$ . Decoupling therefore occurs when  $W_s$  is still relativistic at freeze-out. The resulting density in the regime  $T \gg M$  can be analytically extracted from the Boltzmann equation [28]

$$Y_0 = 0.2 \frac{g_{W_s}}{g_{*s}(T_{dec})}. \quad (4.25)$$

To obtain an estimate for the value of  $\Omega h^2$  we use the expression in eq. (4.20). We first find the current energy density  $\rho_0 = s_{e0} Y_0 M$ , and then divide by the critical energy  $\rho_c$  to arrive at the expression in eq. (4.20). The resulting formula can then be used to obtain a constraint on  $M_{W_s}$

$$M_{W_s} < \frac{\Omega h^2 3.5 \text{ eV}}{Y_0} \simeq (17.7 \text{ eV}) \frac{g_{*s}(T_{dec})}{g_{W_s}} \Omega h^2. \quad (4.26)$$

If we consider a scenario with  $M_{Z'}$  at the TeV scale, with a similar range for the decoupling temperature, the number of the degrees of freedom still relativistic will be of order  $10^2$ . Taking  $\Omega h^2 \sim 1$  we obtain the limit

$$M_{W_s} \lesssim 1 \text{ keV}. \quad (4.27)$$

Such tight constraint is typical of over abundant relativistic WIMP particle, as the condition  $M_{W_s} < M_{Z'}$  forces the singlet to be light because of the suppression of the interactions by a factor  $1/M_{Z'}$ .

- **$M_{W_s} \ll M_{Z'}$  with inflation.**

The constraint in eq. (4.27) is relaxed in the presence of inflation. In this case  $W_s$  is completely diluted by inflation and is regenerated after reheating. In the limit  $M < M_{Z'}$  and  $T_R < M_{Z'}$ , we approximate the  $Z'$ -mediated interaction by a four-point Fermi interaction. In this case  $W_s$  can only annihilate via the diagrams in figures 1 and 2 into matter states, but not those in figures

3 and 4 into gauge bosons and gauginos. The thermal cross-section in the non-relativistic limit  $T_R < M_{W_s}$  is given by

$$\sigma_0 = \frac{32\pi M_{W_s}^2}{M_{Z'}^4} N_{Z'} , \quad (4.28)$$

and by

$$\sigma_0 = \frac{2\pi s}{M_{Z'}^4} N_{Z'} . \quad (4.29)$$

in the relativistic limit  $T_R > M_{W_s}$ . The non-relativistic case is a replication of the analysis for the non-relativistic, out-of-equilibrium production  $W_s$  when  $T_R > M_{Z'}$ . The resulting bound is

$$M_{W_s} > T_R \left( 27 + \frac{1}{2} \ln \left( \frac{M_{W_s}^5}{M_{Z'}^4 T_R} + \frac{M_{W_s}^4}{2 M_{Z'}^4} \right) + \frac{1}{2} \ln (N_{Z'}) \right) . \quad (4.30)$$

In the relativistic limit, the thermal cross section defines a temperature-dependent  $\lambda$  parameter and the integration of the Boltzmann equations requires more care. To proceed we can express the Center of Mass energy of the process as a thermal average, yielding its dependence on the temperature as

$$\langle s \rangle = 4 \langle E^2 \rangle \simeq \left( \frac{5}{4} \right) 40 T^2 . \quad (4.31)$$

The integration of (4.22), with a relativistic equilibrium density, is now at hand and the final constraint on the Wilsonian singlet mass bound takes the form

$$\frac{M_{W_s} T_R^3}{M_{Z'}^4} < 1.6 \times 10^{-28} \left( \frac{g_*^{3/2}}{N_{Z'} g_{eff}^2} \right) . \quad (4.32)$$

As there are three unknown parameters ( $M_{W_s}$ ,  $M_{Z'}$ ,  $T_R$ ) in Eq. (4.32) we cannot infer a definite value for the mass of  $W_s$  and  $Z'$ . We may nevertheless conclude that  $Z'$  should be very heavy.

Finally we remark that direct or indirect dark matter detection of the Wilsonian dark matter candidates will be extremely challenging, as those are expected to be weaker than the prevailing constraints on, say, neutralinos. The reason being that the interaction of the Wilsonian matter states with the Standard Model particles is governed by the  $Z'$  mass scale, which is higher compared to the weak scale that typifies the neutralino interactions. Similarly, indirect detection is suppressed due to the low trapping rate of the Wilsonian singlet matter states in, *e.g.*, the sun [30].

## 5 Conclusions

The issue of dark matter is one of the important enigmas of modern physics. The problem is exacerbated as plausible particle candidates can vary from the ultra-light [2] to the super-massive [4, 29]. It is then sensible to seek guidance from string theory, which is the only contemporary approach that consistently unifies gravity with the gauge interactions.

We emphasise that proposals of dark matter candidates are ample in the literature without reference to their gravitational or string theory connections. Our approach aims to incorporate constraints from string theory, which is a contemporary framework compatible with quantum gravity, in the analysis. In that respect we note that, for example, since the mid eighties there is a plethora of string inspired  $Z'$  studies, whereas, to date, the only known worldsheet construction that allows for an unbroken  $U(1)_{Z'}$  symmetry of the type discussed in the string inspired literature is that of ref. [12]. Wilsonian matter states are a generic consequence of symmetry breaking in string theory by Wilson lines. The main feature which characterises them is the existence of an intrinsic string stabilisation mechanism that forbids their decay to the standard model states. This intrinsic stabilisation mechanism provides the main motivation to study them.

It is then amply rewarding that string constructions indeed contain in them the intrinsic ingredients to produce heavy and stable dark matter. The stabilisation mechanism arises due to the breaking of non-Abelian gauge symmetries in string theory by Wilson line, which gives rise to exotic states that do not satisfy the quantisation conditions of the unbroken non-Abelian gauge symmetry. Well known examples of such states include those that carry fractional electric charge that are highly constrained by experiments. The favoured class of Wilsonian states that can constitute the dark matter are those that are neutral under the Standard Model, but carry fractional charge under an extra  $U(1)_{Z'}$  symmetry. While this possibility has been entertained before in [4], the string derived  $Z'$  model of ref. [12] is the first concrete string derived model in which the Wilsonian singlet dark matter can be realised. This model contains all the ingredients needed to break the  $U(1)_{Z'}$  symmetry at a high or low scale, while maintaining a discrete symmetry that forbids the decay of the Wilsonian singlet matter state. The Wilsonian dark matter singlets in the model of ref. [12] are  $SO(10)$  singlets and arise from the breaking of  $E_6 \rightarrow SO(10) \times U(1)$  by Wilson lines. Even within the constraints of the string construction, as we discussed in section 4, there are varied possibilities depending on the  $U(1)_{Z'}$  breaking scale, the reheating temperature  $T_R$  and the mass of  $W_s$  itself. We can then all but hope that forthcoming experiments will narrow down the possibilities by, for example, discovering a extra vector boson  $Z'$  in the vicinity of the multi-TeV scale.

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